Sample Problems from 2016-2017 National and International Mathematics Contests

2017 MATHCOUNTS State Competition (grades 6-8)

#6 The lengths of the parallel bases of a trapezoid are $7.4 + x$ and $15.6 - x$, where x is a real number. The height of the trapezoid is 12. What is the area of the trapezoid? (Target Round)

#23 The mean age of three students is 20 years. All three of the students are at least 18 years old. What is the greatest possible age of the oldest student, in years? (Countdown Round)

2016 American Mathematics Competition 8 (grade 8 and below)

#9 What is the sum of the distinct prime integer divisors of 2016?

(A) 9 (B) 12 (C) 16 (D) 49 (E) 63

#17 An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1, then how many passwords are possible?

(A) 30 (B) 7290 (C) 9000 (D) 9990 (E) 9999

2017 American Mathematics Competition 10A & 10B (grade 10 and below)

#9 (10B) A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

(A) 1/27 (B) 1/9 (C) 2/9 (D) 7/27 (E) 1/2

2017 American Mathematics Competition 12A and 12B (grade 12 and below)

#11 (12A) Claire adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. She then discovers that she forgot to include one angle. What is the degree measure of the forgotten angle?

(A) 37 (B) 63 (C) 117 (D) 143 (E) 163

2016 American Regions Mathematics League (ARML) – Individual Round

#5 Compute the least possible non-zero value of $A^2 + B^2 + C^2$ such that A, B, and C are integers satisfying $A \log 16 + B \log 18 + C \log 24 = 0$.

2017 American Invitational Mathematics Exam (AIME) (AMC 10 and 12 Honor Roll)

#2 (Contest 1) When each of 702, 787, and 855 is divided by the positive integer m, the remainder is always the positive integer r. When each of 412, 722, and 815 is divided by the positive integer n, the remainder is always the positive integer $s \neq r$. Find $m + n + r + s$.

ANSWERS TO THESE EIGHT PROBLEMS ARE ON THE REVERSE SIDE.

2017 USA Junior Mathematical Olympiad

#3 Let ABC be an equilateral triangle and let P be a point on its circumcircle. Let lines PA and BC intersect at D; let lines PB and CA intersect at E; and let lines PC and AB intersect at F. Prove that the area of triangle DEF is twice the area of triangle ABC.

2107 USA Mathematical Olympiad

#6 Given that a, b, c, and d are nonnegative real numbers such that $a + b + c + d = 4$, find the minimum possible value of $a/(b^3 + 4) + b/(c^3 + 4) + c/(d^3 + 4) + d/(a^3 +4)$.

2016 William Lowell Putnam Mathematical Competition (undergraduate students)

Problem B3 Suppose that S is a finite set of points in the plane such that the area of triangle ΔABC is at most 1 whenever A, B, and C are in S. Show that there exists a triangle of area 4 that (together with its interior) covers the set S.

Modeling Competitions

2017 Mathematical Contest in Modeling (3-person teams of high school or undergraduate students)

Problem C Cooperate and Navigate

Traffic capacity is limited in many regions of the United States due to the number of lanes of roads. For example, in the Greater Seattle area drivers experience long delays during peak traffic hours because the volume of traffic exceeds the designed capacity of the road networks. This is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520, the roads of particular interest for this problem.

Self-driving, cooperating cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads. The behavior of these cars interacting with the existing traffic flow and each other is not well understood at this point.

The Governor of the state of Washington has asked for analysis of the effects of allowing self-driving, cooperating cars on the roads listed above in Thurston, Pierce, King, and Snohomish counties. (See the provided map and Excel spreadsheet). In particular, how do the effects change as the percentage of self-driving cars increases from 10% to 50% to 90%? Do equilibria exist? Is there a tipping point where performance changes markedly? Under what conditions, if any, should lanes be dedicated to these cars? Does your analysis of your model suggest any other policy changes? . . .

2017 Interdisciplinary Contest in Modeling (3-person teams of high school or undergraduate students)

Problem D Optimizing the Passenger Throughput at an Airport Security Checkpoint

Problem F Migration to Mars: Utopian Workforce of the 2100 Urban Society

For details on the above two modeling contests and contest problems, see https://www.comap.com/undergraduate/contests/mcm/

2017 Moody's Mega Math Challenge (3-5 person teams of high school students, plus 1 teacher-coach)

Problem From Sea to Shining Sea: Looking ahead with the National Park Service

For details, see https://m3challenge.siam.org/challenge/about

ANSWERS TO PROBLEMS ON OTHER SIDE:
MATHCOUNTS: #6 (138), #23 (24); AMC 8: #9 (B), #17 (D);
AMC 10B: #9 (D); AMC 12A: #11 (D); ARML: #5 (105); AIME: #2 (062)