

Sample Problems from 2015-2016 National and International Mathematics Contests

2016 MATHCOUNTS State Competition (grades 6-8)

- #4 The areas of three faces of a rectangular prism are 54, 24 and 36 units. What is the length of the space diagonal of this prism? Express your answer in simplest radical form? (Target Round)
- #48 What is the sum of all integer values of n between 75 and 100 such that n divided by 6 has remainder 5 and n divided by 4 has remainder 3? (Countdown Round)

2015 American Mathematics Contest 8 (grade 8 and below)

- #10 How many integers between 1000 and 9999 have four distinct digits?
- (A) 3024 (B) 4536 (C) 5040 (D) 6480 (E) 6561
- #24 A baseball league consists of two four-team divisions. Each team plays every other team in its division N games. Each team plays every team in the other division M games with $N > 2M$ and $M > 4$. Each team plays a 76-game schedule. How many games does a team play within its own division?
- (A) 36 (B) 48 (C) 54 (D) 60 (E) 72

2016 American Mathematics Contests 10A & 10B (grade 10 and below)

- #6 (10A) Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's?
- (A) 13 (B) 26 (C) 102 (D) 103 (E) 110

2016 American Mathematics Contests 12A and 12B (grade 12 and below)

- #6 (12B) All three vertices of triangle ABC lie on the parabola defined by $y = x^2$ with A at the origin and BC parallel to the x-axis. The area of the triangle is 64. What is the length of BC?
- (A) 4 (B) 6 (C) 8 (D) 10 (E) 16

2015 American Regions Mathematics League (ARML) – Individual Round

- #3 A rectangular box has integer edge lengths. The sum of the numerical values of its volume, its surface area, and its twelve edge lengths is 2015. Compute the length of the box's interior diagonal.

2016 American Invitational Mathematics Exam (AIME) (AMC 10 and 12 Honor Roll)

- #5 (Contest 1) Anh read a book. On the first day she read n pages in t minutes, where n and t are positive integers. On the second day Anh read $n+1$ pages in $t+1$ minutes. Each day thereafter Anh read one more page than she read on the previous day, and it took her one more minute than on the previous day until she completely read the 374-page book. It took her a total of 319 minutes to read the book. Find $n + t$.

ANSWERS TO THESE EIGHT PROBLEMS ARE ON THE REVERSE SIDE.

2016 USA Junior Mathematics Olympiad

- #2 Prove that there exists a positive integer $n < 10^6$ such that 5^n has six consecutive zeros in its decimal representation.

2106 USA Mathematics Olympiad

- #4 Find all functions $f: \mathbb{R} \rightarrow \mathbb{R}$ such that for all real numbers x and y ,

$$(f(x) + xy) \cdot f(x - 3y) + (f(y) + xy) \cdot f(3x - y) = (f(x + y))^2.$$

2015 William Lowell Putnam Examination (undergraduate students)

Problem A2 Let $a_0 = 1$, $a_1 = 2$, and $a_n = 4a_{n-1} - a_{n-2}$ for $n > 1$. Find an odd prime factor of a_{2015} .

Modeling Competitions

2016 Mathematical Contest in Modeling (3-person teams of high school or undergraduate students)

A Hot Bath (Problem A). A person fills a bathtub with hot water from a single faucet and settles into the bathtub to cleanse and relax. Unfortunately, the bathtub is not a spa-style tub with a secondary heating system and circulating jets, but rather a simple water containment vessel. After a while, the bath gets noticeably cooler, so the person adds a constant trickle of hot water from the faucet to reheat the bathing water. The bathtub is designed in such a way that when the tub reaches its capacity, excess water escapes through an overflow drain.

Develop a model of the temperature of the bathtub water in space and time to determine the best strategy the person in the bathtub can adopt to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water. Use your model to determine the extent to which your strategy depends upon the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, and the motions made by the person in the bathtub. If the person used a bubble bath additive while initially filling the bathtub to assist in cleansing, how would this affect your model's results?

In addition to the required one-page summary for your MCM submission, your report must include a one-page non-technical explanation for users of the bathtub that describes your strategy while explaining why it is so difficult to get an evenly maintained temperature throughout the bath water.

2016 Interdisciplinary Contest in Modeling (3-person teams of high school or undergraduate students)

Are we heading towards a thirsty planet? (Problem E).

For details on the above two modeling contests, see <https://www.comap.com/undergraduate/contests/mcm/>

2016 Moody's Mega Math Challenge:

Modeling New Approaches to Mobility. For details, see <https://m3challenge.siam.org/challenge/about>

ANSWERS TO PROBLEMS ON OTHER SIDE:

MATHCOUNTS: #4 ($\sqrt{133}$), #48 (178); AMC 8: #10 (B), #24 (B);
AMC 10A: #6 (D); AMC 12B: #6 (C); ARML: #3 ($5\sqrt{19}$); AIME: #5 (053)