

Sample Problems from 2014-2015 National and International Mathematics Contests

2015 MATHCOUNTS State Competition (grades 6-8)

- #4 If $a/(4-a) = b/(5-b) = c/(7-c) = 3$, what is the value of $a + b + c$? (Target Round: 8 problems)
- #25 The product of two nonnegative integers is 10,000. If neither number is a multiple of ten, what is the sum of those two integers? (Countdown Round: 80 problems)

2014 American Mathematics Contest 8 (grade 8 and below)

- #7 There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of the number of girls to the number of boys in her class?
- a) 3:4 b) 4:3 c) 3:2 d) 7:4 e) 2:1
- #16 The "Middle School Eight" basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving the "Middle School Eight" teams?
- a) 60 b) 88 c) 96 d) 144 e) 160

2015 American Mathematics Contests 10A & 10B (grade 10 and below)

- #7 (10B) Consider the operation "minus the reciprocal of," defined by $A*B = A - 1/B$. What is $((1*2)*3) - (1*(2*3))$?
- a) $-7/30$ b) $-1/6$ c) 0 d) $1/6$ e) $7/30$
- #22 (10A) Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?
- a) $47/256$ b) $3/16$ c) $49/256$ d) $25/128$ e) $51/256$

2015 American Mathematics Contests 12A and 12B (grade 12 and below)

- #4 (12A) The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?
- a) $5/4$ b) $3/2$ c) $9/5$ d) 2 e) $5/2$
- #16 (12B) A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?
- a) 18 b) 162 c) $36\sqrt{21}$ d) $18\sqrt{138}$ e) $54\sqrt{21}$

ANSWERS TO THESE EIGHT PROBLEMS ARE ON THE REVERSE SIDE--PLUS MORE SAMPLE PROBLEMS.

2014 American Regions Mathematics League (ARML) – Individual Round

- #2 Let A, B, and C be randomly chosen (not necessarily distinct) integers between 0 and 4 inclusive. Pat and Chris compute the value of $A + B \times C$ by two different methods. Pat follows the proper order of operations, computing $A + (B \times C)$. Chris ignores order of operations, choosing instead to compute $(A + B) \times C$. Compute the probability that Pat and Chris get the same answer.
- #6 Compute the smallest positive integer N such that 2^{14N} and 2014^N have the same number of divisors.

2015 American Invitational Mathematics Exam (AIME) (AMC 10 and 12 Honor Roll)

- #1 (*Contest 1*) The expressions $A = 1 \times 2 + 3 \times 4 + 5 \times 6 + \dots + 37 \times 38 + 39$ and $B = 1 + 2 \times 3 + 4 \times 5 + \dots + 36 \times 37 + 38 \times 39$ are obtained by writing multiplication and addition operators in an alternating pattern between successive integers. Find the positive difference between integers A and B.
- #12 (*Contest 2*) There are $2^{10} = 1024$ possible 10-letter strings in which each letter is either an A or a B. Find the number of such strings that do not have more than 3 adjacent letters that are identical.

2014 William Lowell Putnam Examination (undergraduate students)

Problem A5 (There are 12 problems in all; A1-A6 in the morning, B1-B6 in the afternoon):

Let $P_n(x) = 1 + 2x + 3x^2 + \dots + nx^{n-1}$. Prove that the polynomials $P_j(x)$ and $P_k(x)$ are relatively prime for all positive integers j and k with $j \neq k$.

2015 Mathematical Contest in Modeling (3-person teams of high school or undergraduate students)

Searching for a Lost Plane (Problem B): Recall the lost Malaysian flight MH370. Build a generic mathematical model that could assist "searchers" in planning a useful search for a lost plane feared to have crashed in open water such as the Atlantic, Pacific, Indian, Southern, or Arctic Ocean while flying from Point A to Point B. Assume that there are no signals from the downed plane. Your model should recognize that there are many different types of planes for which we might be searching and that there are many different types of search planes, often using different electronics or sensors. Additionally, prepare a 1-2 page non-technical paper for the airlines to use in their press conferences concerning their plan for future searches.

2015 Interdisciplinary Contest in Modeling (3-person teams of high school or undergraduate students)

Environmental Sustainability Plan (Problem D): This problem involved developing a model for sustainability and a 20-year sustainable development plan for one country on the United Nations Least Developed Countries list. . . Teams searched for pertinent data and grappled with how economic development must consider ecosystem health and social equitability.

(See the enclosed press releases for more information on these two modeling contests and these problems.)

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ANSWERS TO PROBLEMS ON OTHER SIDE:

MATHCOUNTS #4 (12), #25 (641); AMC8 #7 (b), #16 (b); AMC10 #7 (a), #22 (a); AMC12 #4 (b), #16 (c)

ANSWERS TO ARML AND AIME PROBLEMS ON THIS SIDE:

ARML: #2 (9/25), #6 (19133); AIME: #1 (722), #12 (548)