Sample Problems from 2013-2014 National and International Mathematics Contests

2014 American Mathematics Contest 8 (grade 8 and below)

#14 Abe holds 1 green and 1 red jellybean in his hand. Bea holds 1 green, 1 yellow, and 2 red jellybeans in her hand. Each randomly picks a jellybean to show the other. What is the probability that the colors match?

a) 1/4 b) 1/3 c) 3/8 d) 1/2 e) 2/3

#20 A 1 x 2 rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle?

a) $\pi/2$ b) $2\pi/3$ c) π d) $4\pi/3$ e) $5\pi/3$

2014 MATHCOUNTS State Competition Countdown Round (80 problems) (grades 6-8)

#8 How many positive integers divide 6^{2014} but not 6^{2013}?

#60 One way to represent 60 as the sum of consecutive positive integers is 19 + 20 + 21. Including this example, how many ways are there to represent 60 as the sum of two or more consecutive positive integers?

2014 American Mathematics Contests 10A & 10B (grade 10 and below)

#6 (10A) Suppose that A cows give B gallons of milk in C days. At this rate, how many gallons of milk will D cows give in E days?

a) BDE/AC b) AC/BDE c) ABDE/C d) BCDE/A e) ABC/DE

#10 (10B) In the addition shown at right, A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{r} A\,B\,B\,C\,D \\ +\;\underline{B\,C\,A\,D\,A} \\ D\,B\,D\,D\,D \end{array}$$

a) 2 b) 4 c) 7 d) 8 e) 9

2014 American Mathematics Contests 12A and 12B (grade 12 and below)

#3 (12A) Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

a) 2 b) 3 c) 4 d) 5 e) 6

#18 (12B) The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is bad if it is not true that for every N from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to N. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

a) 1 b) 2 c) 3 d) 4 e) 5

ANSWERS TO THESE EIGHT PROBLEMS ARE ON THE REVERSE SIDE.

2014 American Invitational Mathematics Exam (AIME) (AMC 10 and 12 Honor Roll)

#3 (I) Find the number of rational numbers r, $0 < r < 1$, such that when r is written as a fraction in lowest terms, the numerator and denominator have a sum of 1,000.

#9 (II) Ten chairs are arranged in a circle. Find the number of subsets of this set of chairs that contain at least three adjacent chairs.

2013 American Regions Mathematics League (ARML) – Team Round

#4 A palindrome is a positive integer, not ending in 0, that reads the same forwards and backwards. For example, 35253, 171, 44, and 2 are all palindromes, but 17 and 1210 are not. Compute the least positive integer greater than 2013 that cannot be written as the sum of two palindromes.

#8 Compute the sum of all real numbers x such that $[x/2] – [x/3] = x/7$. (Here, $[z]$ denotes the greatest integer $\leq z$.)

2013 William Lowell Putnam Examination (undergraduate students)

Problem A1 (There are 12 problems in all; A1-A6 in the morning, B1-B6 in the afternoon):

> Recall that a regular icosahedron is a convex polyhedron having 12 vertices and 20 faces; the faces are congruent equilateral triangles. On each face of a regular icosahedron is written a nonnegative integer such that the sum of all 20 integers is 39. Show that there are two faces that share a vertex and have the same integer written on them.

2014 Mathematical Contest in Modeling (3-person teams of undergraduate students)

PROBLEM B: College Coaching Legends

Sports Illustrated, a magazine for sports enthusiasts, is looking for the "best all time college coach" male or female for the previous century. Build a mathematical model to choose the best college coach or coaches (past or present) from among either male or female coaches in such sports as college hockey or field hockey, football, baseball or softball, basketball, or soccer. Does it make a difference which time line horizon that you use in your analysis, i.e., does coaching in 1913 differ from coaching in 2013? Clearly articulate your metrics for assessment. Discuss how your model can be applied in general across both genders and all possible sports. Present your model's top 5 coaches in each of 3 different sports.

In addition to the MCM format and requirements, prepare a 1-2 page article for Sports Illustrated that explains your results and includes a non-technical explanation of your mathematical model that sports fans will understand.

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ANSWERS TO PROBLEMS ON OTHER SIDE:
AMC8 #14 (c), #20 (c); MATHCOUNTS #8 (4029), #60 (3 ways); AMC10 #6 (a), #10 (c); AMC12 #3 (b), #18 (b)

ANSWERS TO AIME AND ARML PROBLEMS ON THIS SIDE:
AIME: #3 (200), #9 (581); ARML: #4 (2019), #8 (-21)